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Invasion percolation in a continuum model

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Abstract. A continuum model, where the invasion is determined by the actual local geometry rather than by the common random invasion in lattice models is studied for the first time. Universality is confirmed by the result that the fractal dimension of the cluster is the same as in conventional percolation.

The percolation theory approach to the problem of flow in porous media has been used intensively in recent years (Johnson and Sen 1984). One model which captures the essentials of the flow process, i.e. the displacement of a wetting phase by a non-wetting phase, is known as invasion percolation (Wilkinson and Willemsen 1983, Wilkinson 1986). In this picture it is assumed that the penetration of the 'invading' phase will take place at the 'weakest' (or, the least resistant) 'bond' between the sites which the fluid has already reached and the sites which the fluid has not reached yet. Implementation of this picture was carried out on lattices (Wilkinson 1986, Leclerc and Neale 1988, Wilkinson and Brasony 1984) by attaching a random number to each site on the lattice and by then choosing an origin and letting the cluster expand according to the above rule. Hence, the lowest random number of the cluster's perimeter sites is found and the corresponding site is added to the cluster. One may follow the size of the cluster S (the number of its sites) as a function of the radius of gyration of the cluster R_g and find its fractal dimension d_f, by using the relation (Aharony 1986, Stauffer 1985, Balberg amd Binenbaum 1985)

$$S \sim R_g^{d_f}. \qquad (1)$$

It was found that on lattices the above invasion process yields (Wilkinson 1986, Leclerc and Neale 1988, Wilkinson and Brasony 1984) a cluster which has the same fractal dimension as that of clusters in conventional percolation (Stauffer 1985). In particular $d_f = 1.9$ in two dimensions and $d_f = 2.5$ in three dimensions (Aharony 1986).

The question arises whether the above model yields the same results when applied to a more realistic description of flow in natural or artificial continuum systems. While universal behaviour is usually expected, the possibility of finding a non-universal behaviour even in cluster properties cannot be *a priori* disregarded (Kim *et al* 1987). Following these considerations and the need for a more intuitive and less arbitrary description of the invasion process, we have examined the invasion process in a genuine continuum system (Balberg *et al* 1988). The 'inverted random void' system as used here consists of randomly distributed permeable spheres with a relatively small hard core (which does not alter significantly the conventional percolation threshold, Balberg and Binenbaum 1987). The sphere concentration is well above the conventional percolation threshold (corresponding to an average of $B_c = 4.5$ bonded neighbours in

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two dimensions and $B_c = 2.8$ in three dimensions) (Balberg and Binenbaum 1987). The geometry of the model is illustrated in figure 1. Two spheres in the model are considered bonded if there is an overlap between their soft shells. In our choice of B (the average number of bonds per sphere, see below) all spheres are practically connected and the situation resembles the preliminary lattice site system on which the invasion process is carried out (Wilkinson and Willemsen 1983, Wilkinson 1986, Leclerc and Neale 1988, Wilkinson and Brasony 1984). After the first sphere is picked one searches for the overlapping sphere which is closest to this (the origin) sphere. The bond between these two is then the bond of least resistance (Balberg *et al* 1988) (closest and thus of largest cross section) and it is thus susceptible to invasion. The next stage is to look for the next closest sphere to either one of the two spheres. This sphere becomes the third member of the cluster and the process is repeated. During the process both S and R_g are recorded, where R_g is defined as (Stauffer 1985, Balberg and Binenbaum 1985):

$$R_g = \left(\frac{\sum_{i>j} r_{ij}^2}{S^2} \right)^{1/2} \tag{2}$$

and r_{ij} is the distance between two members of the cluster. In figure 2 we show for illustration the centres of the spheres which form such a two-dimensional invasion percolation cluster.

The simulation procedure was as follows. We implanted randomly (Balberg and Binenbaum 1985, 1987) spheres of radius $a+b$ where a was the hard core radius and b was the soft shell thickness. In all cases we have chosen the ratio $b/a = 0.8$. In two dimensions we have used 8000 disks and in three-dimensions 18 000 spheres. These

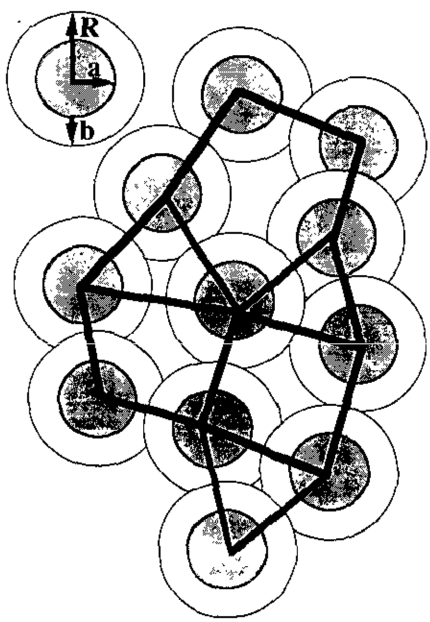

Figure 1. The geometry of the 'inverted random void' system used here as a model of invasion percolation in the continuum. The lines between the spheres centres represent existing bonds. We assume that invasion takes place along the shortest (larger cross section) bonds.

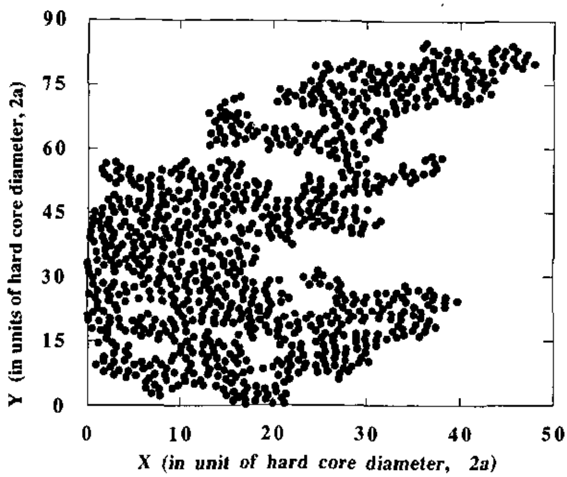

Figure 2. An illustration of a two-dimensional invasion percolation continuum cluster generated in the present work.

concentrations correspond to 1.5 times the conventional percolation threshold concentration in two dimensions and to five times the threshold concentration in three dimensions. We created 20 samples (for each dimension) and for each sample we performed 50 realizations (i.e. we have followed $S(R_g)$ for 50 different origins). The result of the simulations are summarized in figure 3. The average R_g for a given S is given by the data points and the corresponding standard deviations are given by the error bars. The slope of the $\log(S)$ against $\log(R_g)$ plot yields the fractal dimension d_f. These dimensions were found to be $d_f = 1.88$ for the two-dimensional systems and $d_f = 2.7$ for the three-dimensional systems. The regression coefficient calculated for the line connecting the data points is practically 1. These results *are very close* to those obtained for invasion percolation on lattices and in conventional percolation (Aharony 1986, Stauffer 1985, Balberg and Binenbaum 1985). Other averaging procedures yielded

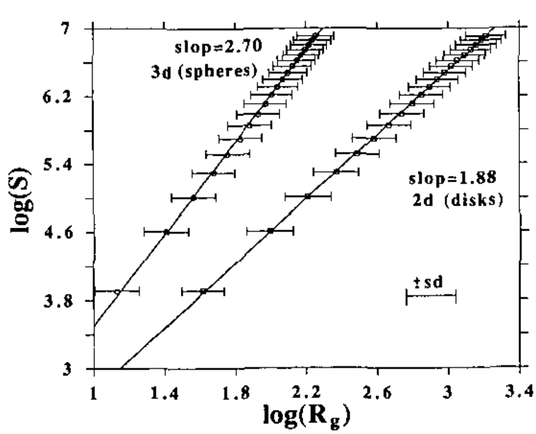

Figure 3. The size of the cluster S, as a function of its gyration radius R_g (measured in units of a, the hard sphere radius). The data points are the averages of 20 samples times 50 realizations and the error bars indicate the standard deviations.

somewhat different results but have confirmed the above conclusion. For example, determining d_f for each realization separately and then carrying the averages yielded $d_f = 1.85 \pm 0.22$ in two dimensions and $d_f = 2.61 \pm 0.42$ in three dimensions. In the averaging scheme some of the slopes were found to yield $d_f > d$ where d is the Euclidean dimension. Since d_f larger than the Euclidean dimension is a result of cases where the origin is on a finite cluster (the cluster 'grows within itself'), we have also calculated the average d_f by disregarding these cases. This procedure yielded the results $d_f = 1.75 \pm 0.14$ in two dimensions and $d_f = 2.48 \pm 0.28$ in three dimensions.

In conclusion, we have suggested that a large cross section between pores is a more realistic definition of a 'weak' bond than the random number attached to a site in the lattice models of invasion percolation. The overall characteristics of this continuum model are, however, similar to those found in lattice invasion percolation. In particular the fractal dimension of the cluster in continuum invasion percolation is the same as in conventional lattice and conventional continuum percolation.

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